Exposure and sensitivity limits related to market risk tolerance

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Summary

- Typical risk tolerance statements imply a target for economic solvency ratios.
- Prudent risk management practice attempts to limit the risk to individual drivers, e.g. equity, interest rate or credit default risk
- Model risk can be reduced by setting in addition sensitivity or exposure limits for individual risk drivers. These can be designed such that they
  - directly link to the risk tolerance
  - adapt in a moderate way to changing market parameters

Part I

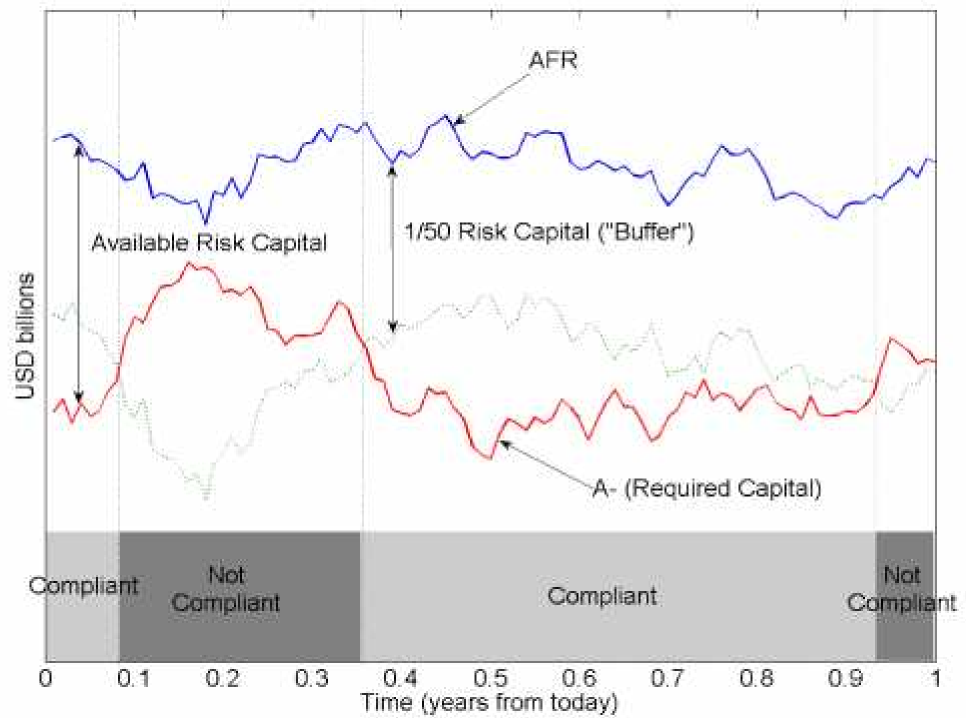
From risk tolerance
to target solvency ratio

A robust risk tolerance statement has to consider the possibility of passive breaches of the basic risk tolerance

Basic statement (B):

(AFR,Risk) must be such that the probability of the economic capital to fall below an A^- requirement within a year does not exceed 1/50

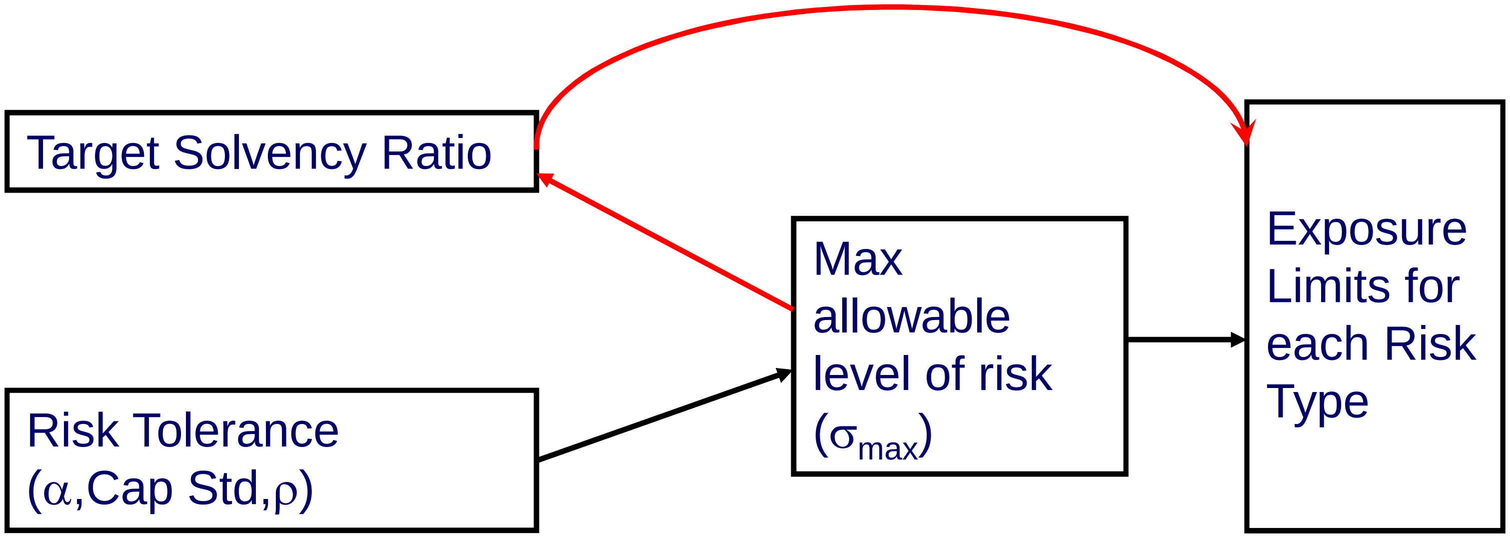
$$\text{AFR} - \text{VaR}(1/50) > \text{VaR}(1/625)$$



Robust statement:

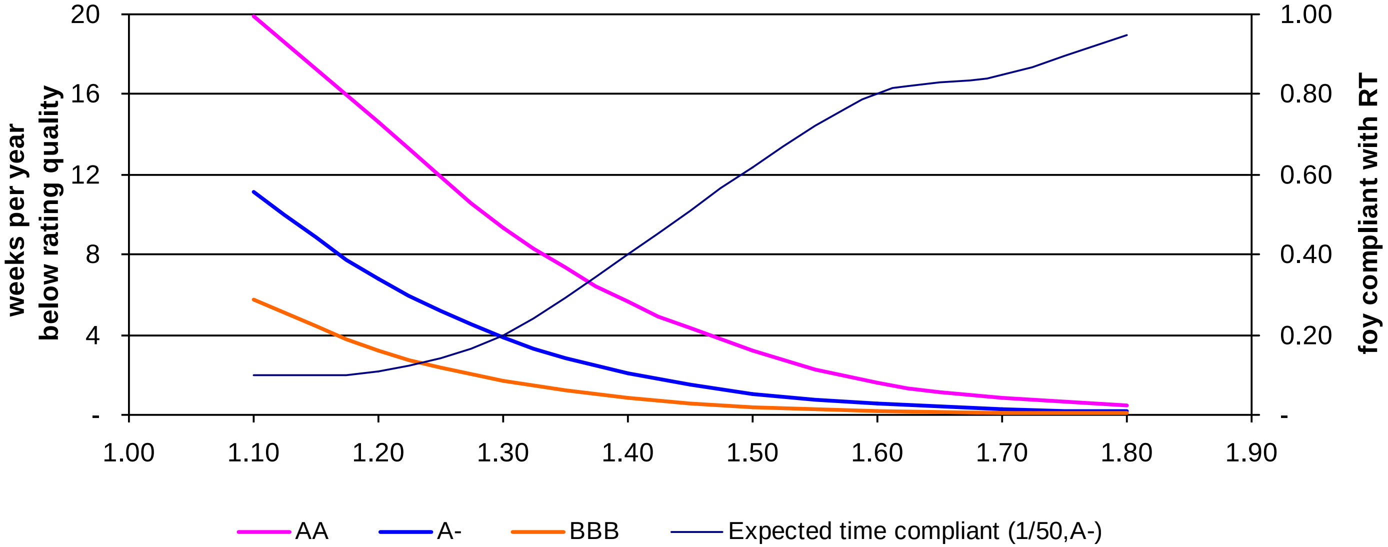
(AFR, Risk) must be such that the *expected amount of time* (q) that statement B holds exceeds r percent of the year

The target solvency capital ratio (SCR) is well suited to formally limit risk tolerance



- SCR and RT both imply a max risk (σ_{max}), hence imply exposure limits
- Many RT statements are compatible with any given SCR
- Limiting the SCR is a convenient way to express risk tolerance quantitatively

relation between SCR, RT and expected time below rating quality



SCR (@ AA standard)	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
Expected time compliant (1/50,A-)	10%	11%	20%	40%	62%	80%	85%	95%

Expected % time below rating boundary *

AA	38.1	28.0	18.0	10.9	6.1	3.1	1.7	0.9
A-	21.4	13.1	7.4	4.0	2.0	1.0	0.5	0.3
BBB	11.1	6.2	3.3	1.6	0.8	0.4	0.2	0.1

* 1.9% corresponds to 1 week per year

Part II

From target solvency ratio
to exposure limits

Define a “safety net” of individual risk driver limits that is linked to the overall risk tolerance

- Step 1: express the risk tolerance as the maximum acceptable difference, L, between mean and α-percentile of the risk distribution, (E – VaR(α)) [.]

- Step 2: dito for individual risk drivers, considering diversification
  - Equities
  - Interest rates
  - Credit default risk

- Step 3: express (E-VaR(α))[.] for a given risk driver by a crude approximation and back solve (E – VaR(α)) [.] < L to yield a limit involving exposure or sensitivities

→ Reduces model risk of the “safety net”

→ Adapts the “safety net” to some changes of market circumstances

Equity limits, a linear case

- Assume full correlation between equities
- Equity exposure is equivalent to N shares of value 1, with effective volatility σ .
- $(E\text{-}VaR(\alpha))[V] = dV/dS \;\; \Delta S_{\alpha} < L$
  $N \, z_{\alpha} \, \sigma < L \;\; \rightarrow \;\; N < L \, / \, (z_{\alpha} \, \sigma)$

→ The volatility, σ, can be calibrated through an economic cycle or at a particular point in time. This choice of depends on the temporal risk horizon and on the market.

Interest rate limits, a quadratic approach

- The interest rate sensitivity of bonds changes with interest rates. Also the probability of observing a certain shift in interest rates depends on their level. This renders problematic the standard practice of limiting the change of value caused by a fixed parallel shift of interest rate.

- Consider 2-nd order Taylor expansion with respect to parallel interest rate movements:

$$\Delta y_{\alpha,t} \cdot |D_t| \cdot MV - \frac{1}{2} \cdot (\Delta y_{\alpha,t})^2 \cdot C_t \cdot MV < L_{\alpha,t}$$

→ Where:

t	= monitoring date
MV	= Market Value at time t
D	= (dMV/dy) / MV, Duration at time t
C	= (d^2MV/dy^2) / MV, Convexity at time t
$\Delta y_{\alpha,t}$	= change in interest rate between t and t + 1 year for a quantile α
$L_{\alpha,t}$	= interest rate risk limit for the group at time t for a quantile α

Interest rate limits, improving the standard delta limit

$$\left| D_t \right| \cdot MV - \frac{1}{2} \cdot \Delta y_{\alpha,t} \cdot C_t \cdot MV < L_{\alpha,t} / \Delta y_{\alpha,t}$$

- Express “delta” limit by the ratio of the maximal tolerated quantile of the loss and the interest rate shift corresponding to this quantile
- Subtract from the “delta” exposure a correction related to the convexity
- The interest rate shift corresponding to the tolerance depends on volatility and level of rates. One may estimate the volatility in many different ways depending on the exact purpose of the limits.

Linking credit concentration risk to a VaR limit (1/3)

- The purpose of a credit concentration limit is to manage the risk of default events. The exposure given default, aggregated across individual issuers, is given by

$$EGD = N\,(\,P_D - (\,1 - LGD)) < L_\alpha$$

Where		
	N	is the notional
	P_D	is the dirty price
	LGD	is the loss given default
	L_α	is the tolerated loss amount

- If the probability of default, p_d, is equal to the confidence level, α, applied to determine the tolerated loss amount, the exposure limit is

$$N < L_\alpha \,/\, (\,P_D - (\,1 - LGD))$$

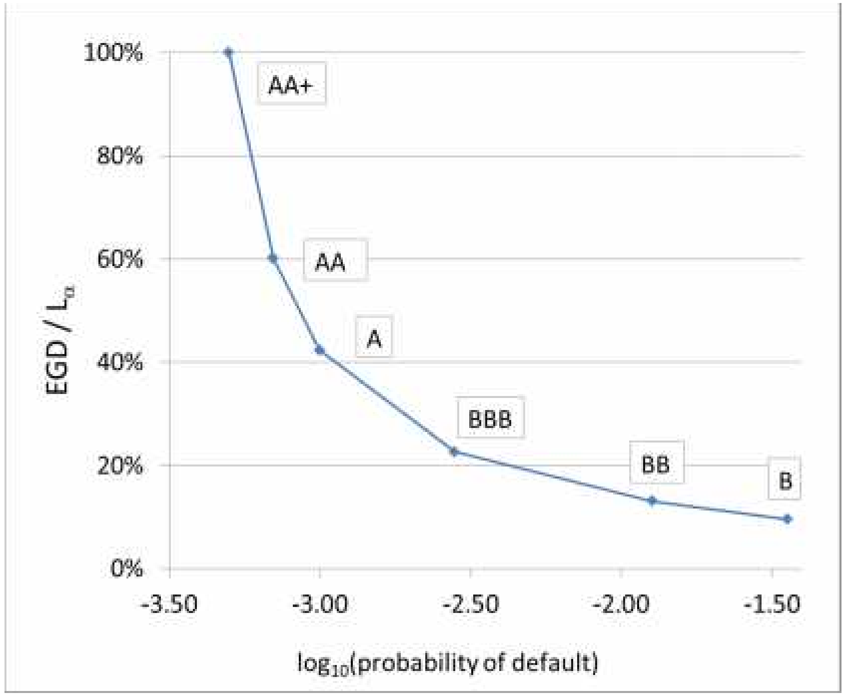
E.g. $\alpha = 1/2000$ and the counterparty is rated AA with $p_d = 1/2000$

- If p_d is smaller than the confidence level the same limit applies

Linking credit concentration risk to a VaR limit (2/3)

- If the probability of default is larger than the confidence level, construct scenarios of several bonds defaulting that has a probability of default equal to the confidence level
- By splitting the tolerated loss amount equally between bonds of the same rating, exposure limits can be constructed easily
- Key inputs for this exercise are the default probabilities and the dependence structure assumed.

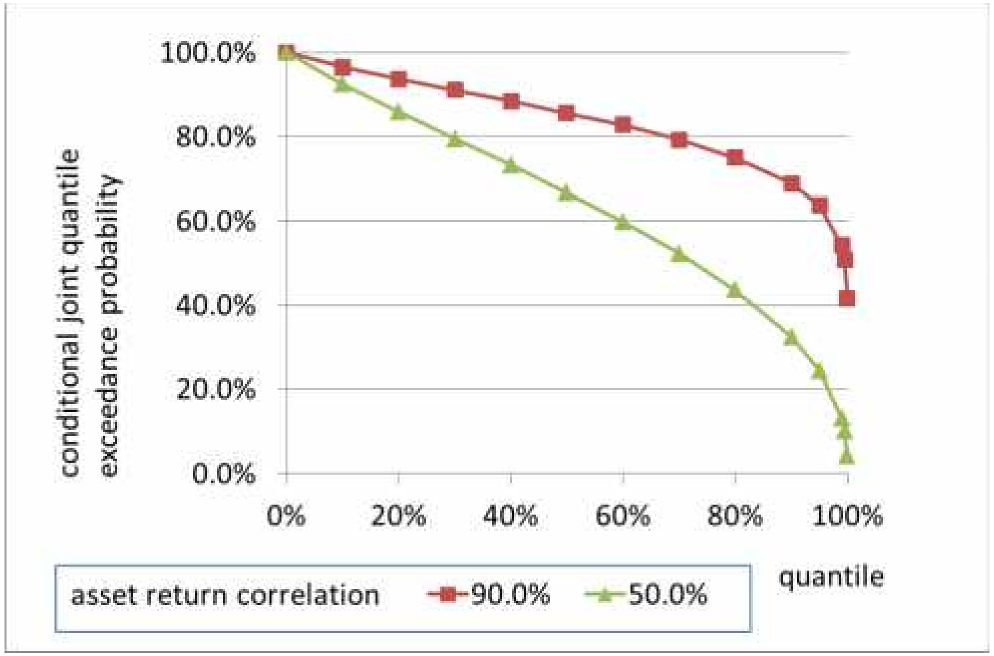
- Example:
  > rank bonds in rating band per size
  > default of largest bond with probability of default, p_d, corresponding to rating.
  > Further defaults with probability equal to correlation plus p_d
  > Assumed stressed asset return correlation of 90%.



Dependence modeling is essential. (3/3)

Recommend stressed asset return correlations to account for crisis conditions

- The Merton approach is widely used in modeling joint defaults.
- Conditional joint quantile exceedance probability, JQEP, quantifies dependence between joint default events. Due to the asymptotic independence of the Gauss copula it is much lower than the asset correlation.
- Stressing the asset correlation increases materially the JQEP



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